2D Wave eqn...

$$U_{tt} = c^2 (U_{xx} + U_{yy})$$

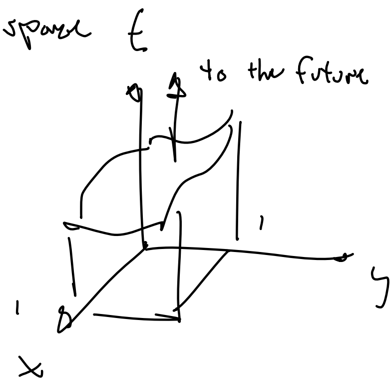
$0 \le x \le 1$

$0 \le y \le 1$

$0 \le t$

(all x, y, t are dimensionless)

We want $U(x, y, t)$ in x-y-t space



Use spacings of $\Delta x, \Delta y, \Delta t$

and finite dif. method

Notation $U(x_i, y_j, z_k)$

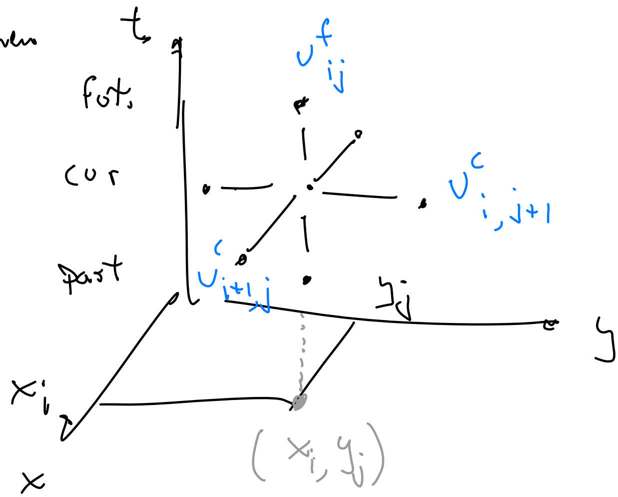
$= U_{i,j,k}$

or better

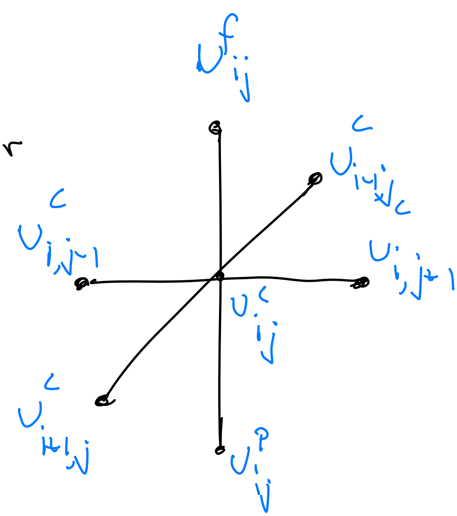
$U_{future}(x_i, y_j) = U^f_{ij}$ $\quad U_{current}(x_i, y_j) = U^c_{ij}$

and $U_{past}(x_i, y_j) = U^p_{ij}$

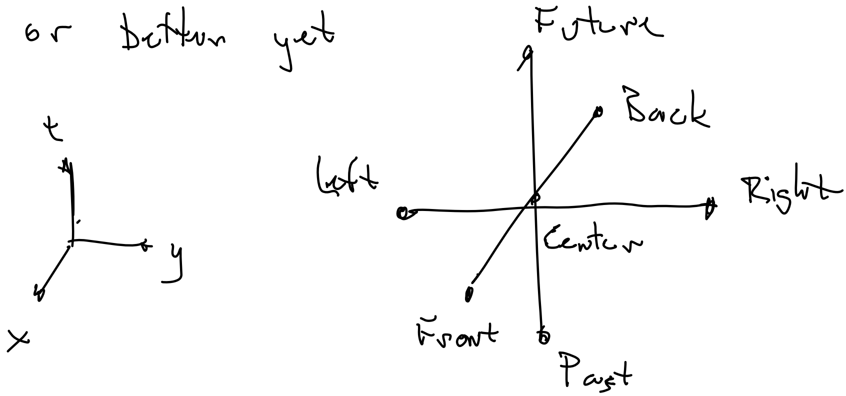
then



or



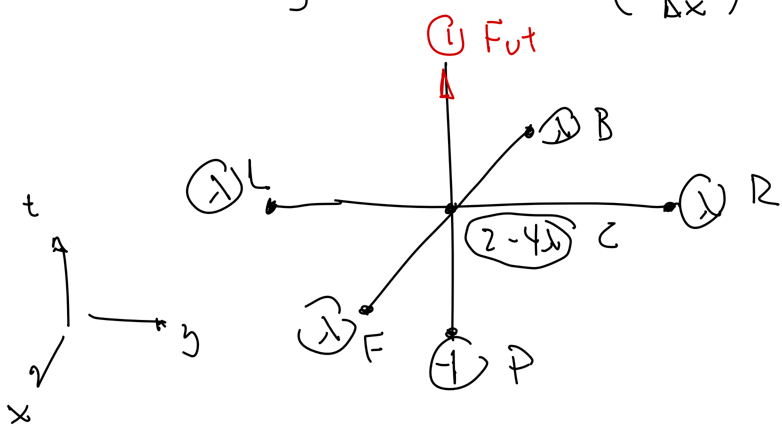
or better yet



Then $u_{tt} = c^2(u_{xx} + u_{yy})$ becomes

$$\frac{Fut - 2C + P}{\Delta t^2} = c^2\left[\frac{(F - 2C + B)}{\Delta x^2} + \frac{(L - 2C + R)}{\Delta y^2}\right]$$

if $\Delta x = \Delta y$ and $\lambda = \left(\frac{c\,\Delta t}{\Delta x}\right)^2$ then solving for Fut,



or

$$Fut = \lambda(L + R + F + B) + (2 - 4\lambda)C - P$$

is valid for $t_k \geq 1\,\Delta t$ (2 previous time layers are known – you know the "current" and "past" layers)

But for the first unknown time layer you also need the initial conditions

$$U\Big|_{t=0} = f(x,y) \quad \text{(initial displacement given)}$$

$$\frac{\partial U}{\partial t}\Big|_{t=0} = 0 \quad \text{(zero speed)}$$

This translates to

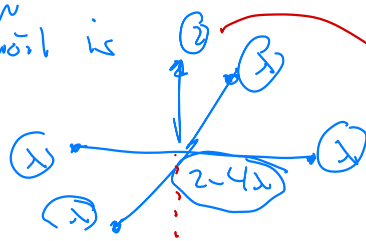
Fut

Current (initial time)

Phantom

$$\frac{Fut - Phantom}{2\Delta t} = 0 \Rightarrow Fut = Phan$$

So the stencil is



at time $t = 1\Delta t$

at time $t=0$

nothing here any more!